EPFL

EXAMEN MA1-3
JANUARY XXTH 202X

MSE 487 – Mathematical Methods for Materials Science

NAME :	Grade : / XX

Rules and Instructions

Please write your name first thing on top of this page.

You are not allowed to use any document, computer, tablet, phone, calculator, or any connected object. An exam sheet with all the important formulae and definitions is given with the exam.

Please write clearly on this response sheet. Fill-in the boxes where to put the responses, and put your demonstration in the allocated spaces.

If you need more space for your developments, you can use draft sheets provided and indicate clearly the question number. You can attach it to your exam, staplers will be available at the end of the exam.

Also, if you did not have time to transfer your response from your draft to the response sheet during the allocated time, you can attach your draft to your exam. Indicate clearly which exercise / question is addressed on the draft.

All exercises, and the vast majority of individual questions, can be addressed independently.

Have a good exam!

Exercise 1 : Fourier transform of the wave equation **/ 10**

We consider an electric field $E(x,t)$ of an electromagnetic wave linearly polarized and traveling along the x direction. $E(x,t)$ is a function of position x and time t. It is continuous and differentiable, and so are its partial derivatives of the first and second order. The wave equation governing this field is given by:

$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

1a. Assuming that it exists, taking the Fourier transform over the time variable t: $\hat{E}(x,\omega) = \int_{-\infty}^{+\infty} E(x,t)e^{-i\omega t}dt$, show that: $\frac{\partial^2 \hat{E}}{\partial x^2} = \frac{-\omega^2}{c^2}\hat{E}$ (*2 pts*)

Demonstration :

1b. Deduce that one can write: $\hat{E}(x,\omega) = Ae^{-ikx} + Be^{ikx}$, with $k = \frac{\omega}{c}$. (*2 pts*)

Demonstration :

1c. Are A and B: (*1 pt*)

Constants: ☐ Functions of x and ω ☐ Functions of ω only ☐ Functions of x only: ☐

1d. Taking the inverse Fourier transform expressed as $E(x,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{E}(x,\omega)e^{i\omega t}d\omega$, show that the general solution of the wave equation can be written as:

$$E(x,t) = f\left(t - \frac{x}{c}\right) + g\left(t + \frac{x}{c}\right)$$

Where f and g are two functions defined by an integral. (*3 pts*)

Demonstration :

1e. We consider the particular case where $A = A_0\delta(\omega + \omega_0)$ and $B = 0$, where A_0 is a constant and δ is the Dirac delta function.

Show that $E(x,t) = E_0 e^{-i(\omega_0 t - k_0 x)}$ (with $k_0 = \frac{\omega_0}{c}$), and express E_0 as a function of A_0: $E_0 =$ (*2 pts*)

Demonstration :